Semester-IV
MPHYEC-1
Unit-1
Scattering theory

Compiled by

Dr. Ashok Kumar Jha

Department of Physics

Patna University

Mob: 7903067108

E-mail: ashok.jha1984@gmail.com

Based on: http://www.tcm.phy.cam.ac.uk/~bds10/aqp/lec20-21_compressed.pdf

Scattering fundamentals

- Scattering can be broadly defined as the *redirection of radiation out of the original direction of propagation*, usually due to interactions with molecules and particles
- Reflection, refraction, diffraction etc. are actually all just forms of scattering
- Matter is composed of discrete electrical charges (atoms and molecules – dipoles)
- Light is an oscillating EM field – excites charges, which radiate EM waves
- These radiated EM waves are *scattered waves*, excited by a source external to the scatterer
- The *superposition of incident and scattered EM waves*is what is observed

Scattering theory is important as it underpins one of the most ubiquitous tools in physics.

- Almost everything we know about nuclear and atomic physics has been discovered by scattering experiments,

  e.g. Rutherford's discovery of the nucleus, the discovery of sub-atomic particles (such as quarks), etc.

- In low energy physics, scattering phenomena provide the standard tool to explore solid state systems,

  e.g. neutron, electron, x-ray scattering, etc.

- As a general topic, it therefore remains central to any advanced course on quantum mechanics.

Source: http://www.tcm.phy.cam.ac.uk/~bds10/aqp/lec20-21_compressed.pdf

When does scattering matter?

- Scattering can be ignored whenever gains in intensity due to scattering along a line of sight are negligible compared to:
  - Losses due to extinction
  - Gains due to thermal emission
- Usually satisfied in the thermal IR band and for microwave radiation when no precipitation (rain, snow etc.) is present
- Also can be ignored when considering direct radiation from a point source, such as the sun
- In the UV, visible and near-IR bands, scattering is the dominant source of radiation along any line of sight, other than that looking directly at the sun

Types of scattering

- Elastic scattering – the wavelength (frequency) of the scattered light is the same as the incident light (*Rayleigh and Mie scattering*)

- Inelastic scattering – the emitted radiation has a wavelength different from that of the incident radiation (*Raman scattering*, *fluorescence*)

- Quasi-elastic scattering – the wavelength (frequency) of the scattered light shifts (e.g., in moving matter due to Doppler effects)

Types of scattering

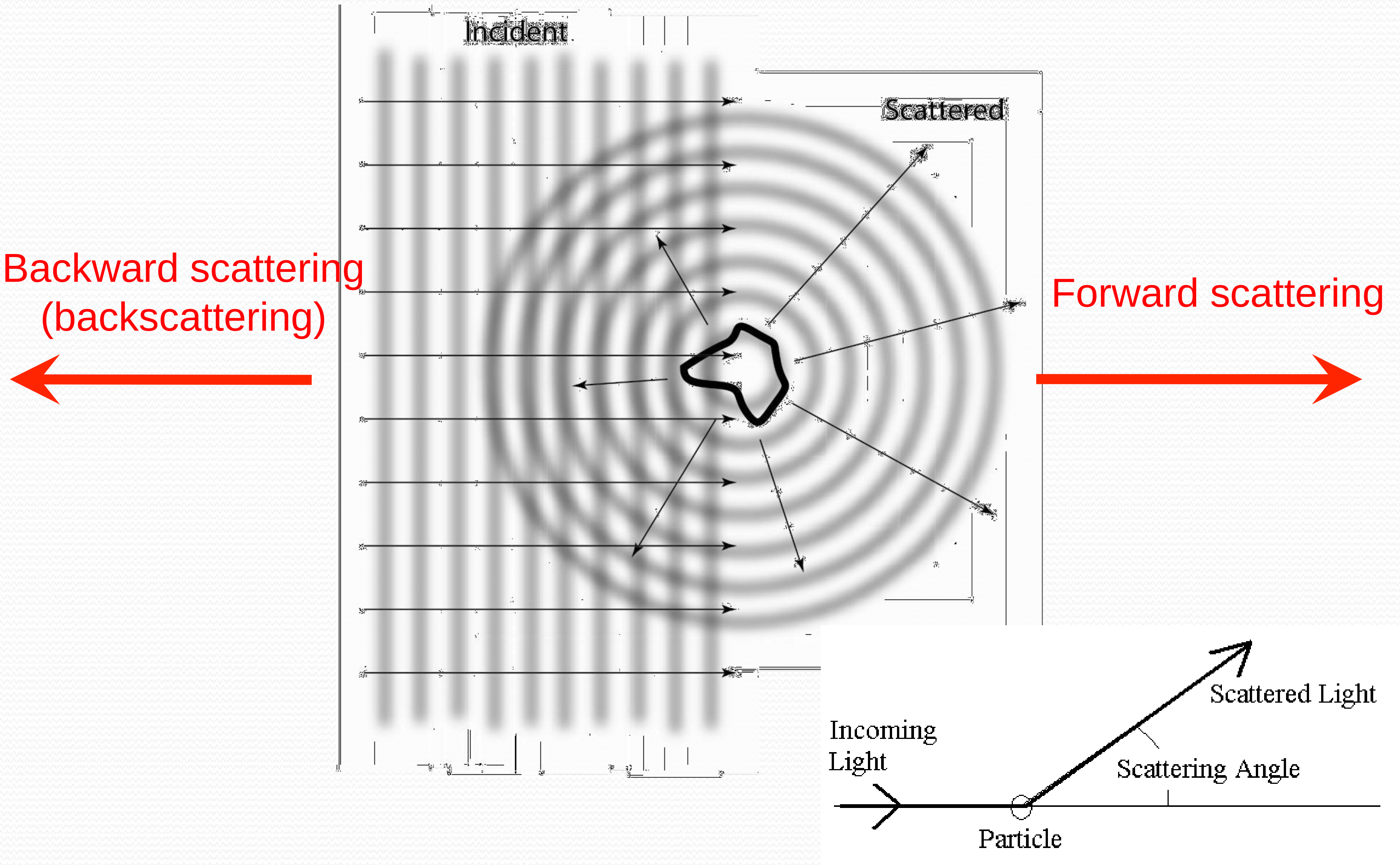
- In an idealized scattering experiment, a sharp beam of particles (A) of definite momentum **k** are scattered from a localized target (B).
- As a result of collision, several outcomes are possible:

$$A + B \longrightarrow \begin{cases} A + B & \text{elastic} \\ A + B^* \\ A + B + C \end{cases} \text{inelastic} \\ \quad\; C \quad \text{absorption}$$

- In high energy and nuclear physics, we are usually interested in deep inelastic processes.
- To keep our discussion simple, we will focus on **elastic processes** in which both the energy and particle number are conserved – although many of the concepts that we will develop are general.

Source: http://www.tcm.phy.cam.ac.uk/~bds10/aqp/lec20-21_compressed.pdf

Scattering geometry



Parameters governing scattering

1. The wavelength (λ) of the incident radiation

2. The size of the scattering particle, usually expressed as the non-dimensional size parameter, x:

$$x = \frac{2\pi r}{\lambda}$$

r is the radius of a spherical particle, **λ** is wavelength

3. The particle optical properties relative to the surrounding medium: the complex refractive index

4. Scattering regimes:

- **x << 1** : Rayleigh scattering
- **x ~ 1** : Mie scattering
- **x >>1** : Geometric scattering

Differential Cross Section

Both classical and quantum mechanical scattering phenomena are characterized by the scattering cross section, σ.

- Consider a collision experiment in which a detector measures the number of particles per unit time, $N\,d\Omega$, scattered into an element of solid angle $d\Omega$ in direction (θ, ϕ).
- This number is proportional to the incident flux of particles, j_I defined as the number of particles per unit time crossing a unit area normal to direction of incidence.
- Collisions are characterised by the **differential cross section** defined as the ratio of the number of particles scattered into direction (θ, ϕ) per unit time per unit solid angle, divided by incident flux,

$$\frac{d\sigma}{d\Omega} = \frac{N}{j_I}$$

Source: http://www.tcm.phy.cam.ac.uk/~bds10/aqp/lec20-21_compressed.pdf

Cross Section

- From the differential, we can obtain the **total cross section** by integrating over all solid angles

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \, \sin\theta \frac{d\sigma}{d\Omega}$$

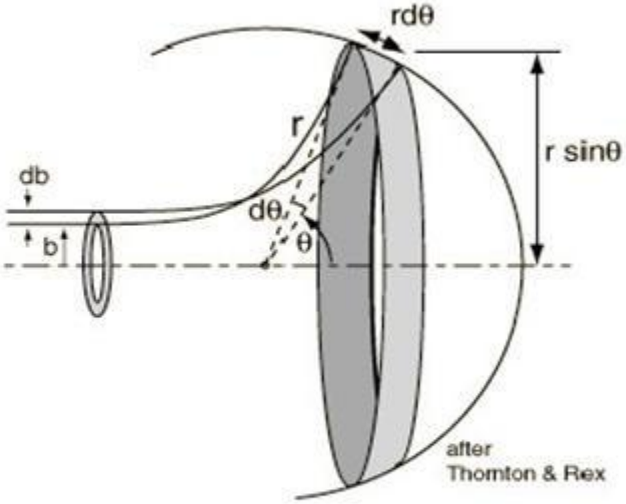
- The cross section, which typically depends sensitively on energy of incoming particles, has dimensions of area and can be separated into σ_{elastic}, $\sigma_{\text{inelastic}}$, σ_{abs}, and σ_{total}.
- In the following, we will focus on elastic scattering where internal energies remain constant and no further particles are created or annihilated,

  e.g. low energy scattering of neutrons from protons.
- However, before turning to quantum scattering, let us consider classical scattering theory.

Source: http://www.tcm.phy.cam.ac.uk/~bds10/aqp/lec20-21_compressed.pdf

Classical Theory

- In classical mechanics, for a central potential, $V(r)$, the angle of scattering is determined by **impact parameter** $b(\theta)$.
- The number of particles scattered per unit time between θ and $\theta + d\theta$ is equal to the number incident particles per unit time between b and $b + db$.
- Therefore, for incident flux j_{I}, the number of particles scattered into the solid angle $d\Omega = 2\pi \sin\theta \, d\theta$ per unit time is given by



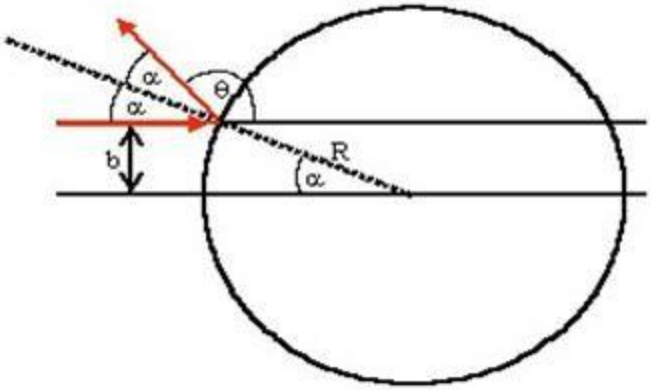
$$N \, d\Omega = 2\pi \sin\theta \, d\theta \, N = 2\pi b \, db \, j_{\mathrm{I}}$$

$$\text{i.e.} \quad \frac{d\sigma(\theta)}{d\Omega} \equiv \frac{N}{j_{\mathrm{I}}} = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right|$$

Source: http://www.tcm.phy.cam.ac.uk/~bds10/aqp/lec20-21_compressed.pdf

Classical Theory

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$



- For elastic scattering from a hard (impenetrable) sphere,

$$b(\theta) = R\sin\alpha = R\sin\left(\frac{\pi-\theta}{2}\right) = -R\cos(\theta/2)$$

- As a result, we find that $\left|\frac{db}{d\theta}\right| = \frac{R}{2}\sin(\theta/2)$ and

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{R^2}{4}$$

- As expected, total scattering cross section is just $\int d\Omega \frac{d\sigma}{d\Omega} = \pi R^2$, the projected area of the sphere.

Source: http://www.tcm.phy.cam.ac.uk/~bds10/aqp/lec20-21_compressed.pdf

Classical Theory

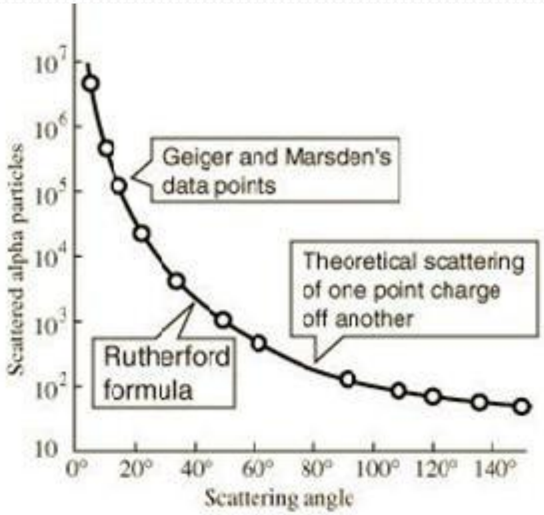
- For **classical Coulomb scattering**,

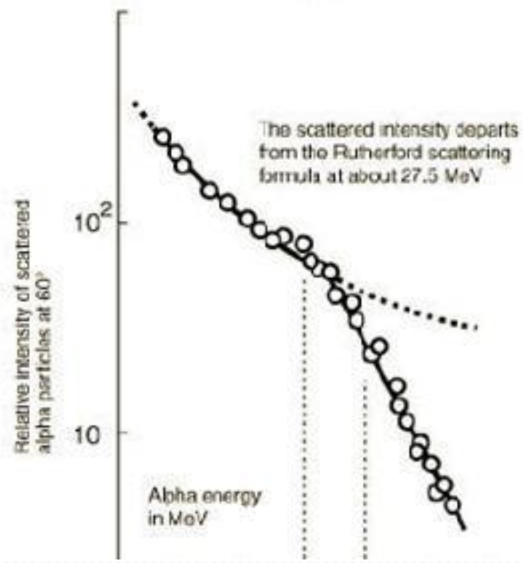
$$V(r) = \frac{\kappa}{r}$$

particle follows hyperbolic trajectory.

- In this case, a straightforward calculation obtains the **Rutherford formula:**

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{\kappa^2}{16E^2}\frac{1}{\sin^4\theta/2}$$

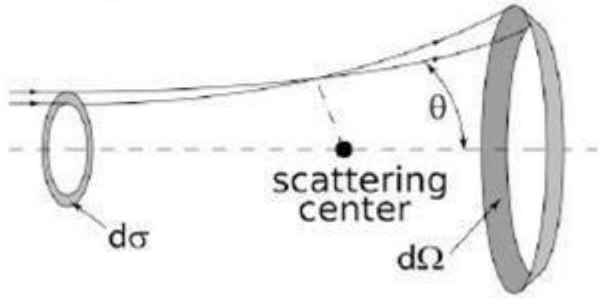




Source: http://www.tcm.phy.cam.ac.uk/~bds10/aqp/lec20-21_compressed.pdf

Quantum Theory

- Simplest scattering experiment: plane wave impinging on localized potential, $V(\mathbf{r})$, e.g. electron striking atom, or α particle a nucleus.
- Basic set-up: flux of particles, all at the same energy, scattered from target and collected by detectors which measure angles of deflection.



- In principle, if all incoming particles represented by wavepackets, the task is to solve time-dependent Schrödinger equation,

$$i\hbar\,\partial_t \Psi(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right]\Psi(\mathbf{r},t)$$

and find probability amplitudes for outgoing waves.

Source: http://www.tcm.phy.cam.ac.uk/~bds10/aqp/lec20-21_compressed.pdf

Quantum Theory

- However, if beam is "switched on" for times long as compared with "encounter-time", steady-state conditions apply.
- If wavepacket has well-defined energy (and hence momentum), may consider it a plane wave: $\Psi(\mathbf{r}, t) = \psi(\mathbf{r}) e^{-iEt/\hbar}$.
- Therefore, seek solutions of time-*independent* Schrödinger equation,

$$E\psi(\mathbf{r}) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r})$$

subject to boundary conditions that incoming component of wavefunction is a plane wave, $e^{i\mathbf{k}\cdot\mathbf{r}}$ (cf. 1d scattering problems).

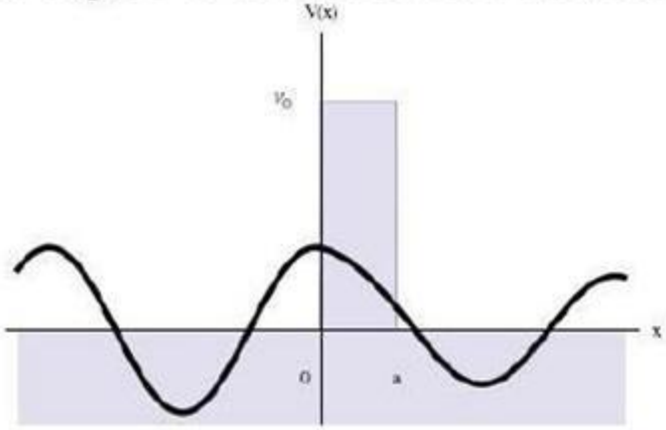
- $E = (\hbar \mathbf{k})^2/2m$ is energy of incoming particles while flux given by,

$$\mathbf{j} = -i\frac{\hbar}{2m}(\psi^* \nabla \psi - \psi \nabla \psi^*) = \frac{\hbar \mathbf{k}}{m}$$

Source: http://www.tcm.phy.cam.ac.uk/~bds10/aqp/lec20-21_compressed.pdf

Description in one dimension

- In one-dimension, interaction of plane wave, e^{ikx}, with localized target results in degree of reflection and transmission.

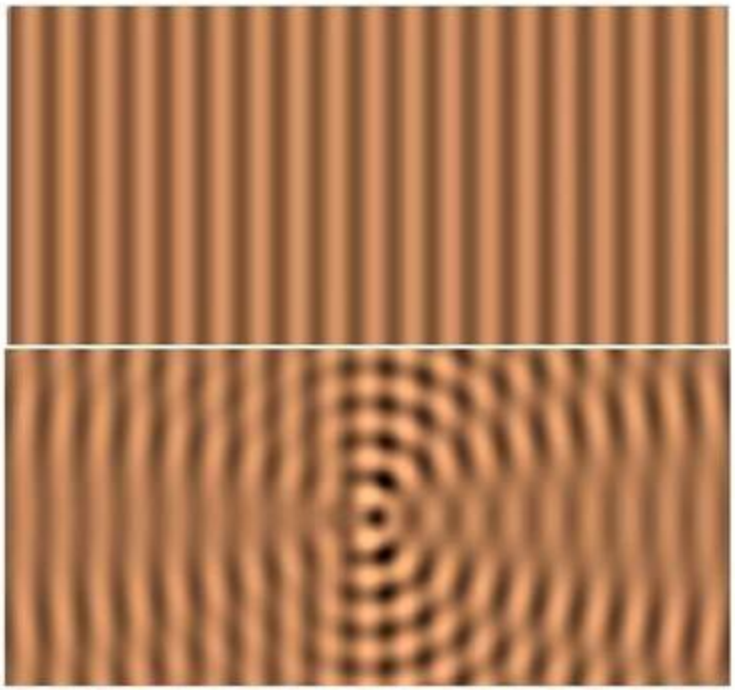


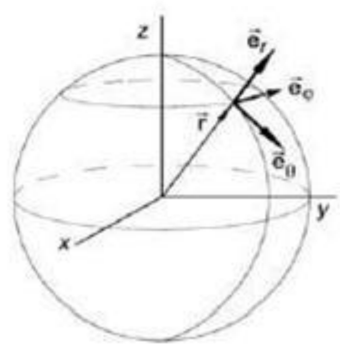
- Both components of outgoing scattered wave are plane waves with wavevector $\pm k$ (energy conservation).
- Influence of potential encoded in **complex amplitude** of reflected and transmitted wave – fixed by time-independent Schrödinger equation subject to boundary conditions (flux conservation).

Source: http://www.tcm.phy.cam.ac.uk/~bds10/aqp/lec20-21_compressed.pdf

More than one dimension

- In higher dimension, phenomenology is similar – consider plane wave incident on localized target:

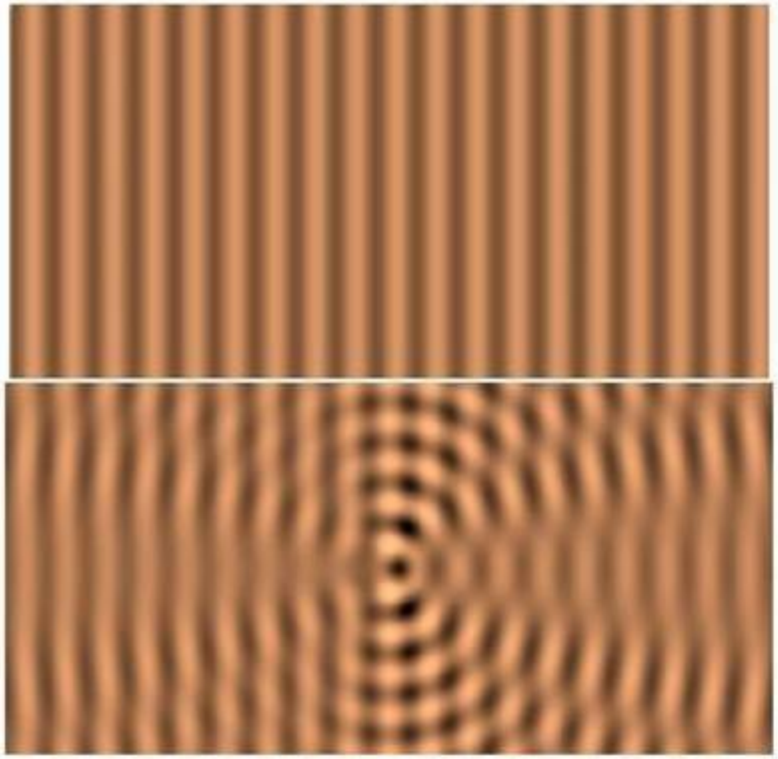




- Outside localized target region, wavefunction involves superposition of incident plane wave and scattered (spherical wave)

Source: http://www.tcm.phy.cam.ac.uk/~bds10/aqp/lec20-21_compressed.pdf

More than one dimension



- If we define z-axis by $\mathbf{k}$ vector, plane wave can be decomposed into superposition of incoming and outgoing spherical wave:
- If $V(r)$ isotropic, short-ranged (faster than $1/r$), and **elastic** (particle/energy conserving), scattering wavefunction given by,

Source: http://www.tcm.phy.cam.ac.uk/~bds10/aqp/lec20-21_compressed.pdf

THANK YOU ALL